HW Solutions to Section 9.7: 4, 5, 6, 8, 9, 10. (Xi Li’ Section)

4. (a) \[ x = 1 \cos \pi = -1, \quad y = 1 \sin \pi = 0, \quad z = e, \]
the point is \((-1, 0, e)\) in rectangular coordinates.

(b) \[ x = 1 \cos \frac{3\pi}{2} = 0, \quad y = 1 \sin \frac{3\pi}{2} = -1, \quad z = 2, \]
so the point is \((0, -1, 2)\) in rectangular coordinates.

5. (a) \[ r^2 = x^2 + y^2 = 1^2 + (-1)^2 = 2 \] so \( r = \sqrt{2}; \) \( \tan \theta = \frac{y}{x} = \frac{-1}{1} = -1 \) and the point \((1, -1)\) is in the fourth quadrant of the \(xy\)-plane, so \( \theta = \frac{3\pi}{4} + 2\pi; \) \( z = 4. \) Thus, one set of cylindrical coordinates is \((\sqrt{2}, \frac{3\pi}{4}, 4)\).

(b) \[ r^2 = (-1)^2 + (-\sqrt{3})^2 = 4 \] so \( r = 2; \) \( \tan \theta = \frac{-\sqrt{3}}{1} = -\sqrt{3} \) and the point \((-1, -\sqrt{3})\) is in the third quadrant of the \(xy\)-plane, so \( \theta = \frac{5\pi}{3} + 2\pi; \) \( z = 2. \) Thus, one set of cylindrical coordinates is \((2, \frac{5\pi}{3}, 2)\).

6. (a) \[ r^2 = x^2 + y^2 = 3^2 + 3^2 = 18 \] so \( r = \sqrt{18} = 3\sqrt{2}; \) \( \tan \theta = \frac{y}{x} = \frac{3}{3} = 1 \) and the point \((3, 3)\) is in the first quadrant of the \(xy\)-plane, so \( \theta = \frac{\pi}{4} + 2\pi; \) \( z = -2. \) Thus, one set of cylindrical coordinates is \((3\sqrt{2}, \frac{\pi}{4}, -2)\).
8. (a) 
\[ x = 5 \sin \frac{\pi}{2} \cos \pi = -5, \quad y = 5 \sin \frac{\pi}{2} \sin \pi = 0, \]
\[ z = 5 \cos \frac{\pi}{2} = 0 \]
so the point is \((-5, 0, 0)\) in rectangular coordinates.

8. (b) 
\[ x = 4 \sin \frac{\pi}{4} \cos \frac{3\pi}{4} = 4 \left( \frac{\sqrt{2}}{2} \right) \left( -\frac{\sqrt{2}}{2} \right) = -\sqrt{6}, \]
\[ y = 4 \sin \frac{\pi}{4} \sin \frac{3\pi}{4} = 4 \left( \frac{\sqrt{2}}{2} \right) \left( \frac{\sqrt{2}}{2} \right) = \sqrt{6}, \]
\[ z = 4 \cos \frac{\pi}{4} = 4 \left( \frac{\sqrt{2}}{2} \right) = 2 \]
so the point is \((-\sqrt{6}, \sqrt{6}, 2)\) in rectangular coordinates.

9. (a) \[ \rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{1 + 3 + 12} = 4, \quad \cos \phi = \frac{z}{\rho} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \phi = \frac{\pi}{6}, \quad \text{and} \]
\[ \cos \theta = \frac{x}{\rho \sin \phi} = \frac{1}{4 \sin(\pi/6)} = \frac{1}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{3} \] (since \(y > 0\)). Thus spherical coordinates are \(\left(4, \frac{\pi}{3}, \frac{\pi}{6}\right)\).

(b) \[ \rho = \sqrt{0 + 1 + 1} = \sqrt{2}, \quad \cos \phi = \frac{-1}{\sqrt{2}} \quad \Rightarrow \quad \phi = \frac{3\pi}{4}, \quad \text{and} \quad \cos \theta = \frac{0}{\sqrt{2} \sin(3\pi/4)} = 0 \quad \Rightarrow \quad \theta = \frac{3\pi}{2} \] (since \(y < 0\)). Thus spherical coordinates are \(\left(\sqrt{2}, \frac{3\pi}{2}, \frac{3\pi}{4}\right)\).

10. (a) \[ \rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{0 + 3 + 1} = 2, \quad \cos \phi = \frac{z}{\rho} = \frac{1}{2} \quad \Rightarrow \quad \phi = \frac{\pi}{3}, \quad \text{and} \quad \cos \theta = \frac{x}{\rho \sin \phi} = \frac{0}{2 \sin(\pi/3)} = 0 \quad \Rightarrow \]
\[ \theta = \frac{\pi}{2} \] (since \(y > 0\)). Thus spherical coordinates are \(\left(2, \frac{\pi}{2}, \frac{\pi}{3}\right)\).

(b) \[ \rho = \sqrt{1 + 1 + 6} = 2 \sqrt{2}, \quad \cos \phi = \frac{\sqrt{3}}{2 \sqrt{2}} = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \phi = \frac{\pi}{6}, \quad \text{and} \quad \cos \theta = \frac{-1}{2 \sqrt{2} \sin(\pi/6)} = -\frac{1}{\sqrt{2}} \quad \Rightarrow \quad \theta = \frac{3\pi}{4} \] (since \(y > 0\)). Thus spherical coordinates are \(\left(2 \sqrt{2}, \frac{3\pi}{4}, \frac{\pi}{6}\right)\).