Use implicit differentiation to find the second derivative

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I give one example concerning how to use implicit differentiation to find the second derivative. I will not cover that in class. However, it may appear in the midterm.

Q: Find $\frac{d^2 y}{dx^2}$ where $y^2 - xy = 8$

Solution: We first find the first order derivative: differentiate the above equation directly, we obtain:

$$2y \frac{dy}{dx} - (y + x \frac{dy}{dx}) = 0$$

Rearranging and solve the equation for $\frac{dy}{dx}$, we get:

$$\frac{dy}{dx} = \frac{y}{2y - x} \quad (0.1)$$

To find the second derivative, we differentiate (0.1) directly, using quotient rule:

$$\frac{d^2 y}{dx^2} = \frac{\frac{du}{dx}(2y - x) - y(2\frac{dy}{dx} - 1)}{(2y - x)^2}$$

Using the first derivative (0.1), we have

$$\frac{d^2 y}{dx^2} = \frac{\frac{dy}{dx}(2y - x) - y(2\frac{dy}{dx} - 1)}{(2y - x)^2}$$

$$= \frac{\frac{y}{2y - x}(2y - x) - y(2\frac{2y - x}{2y - x} - 1)}{(2y - x)^2}$$

$$= \frac{y - \frac{2y^2}{2y - x} + y}{(2y - x)^2}$$

$$= \frac{2y - \frac{2y^2}{2y - x}}{(2y - x)^2}$$

$$= \frac{2y(2y - x) - 2y^2}{(2y - x)^3}$$

$$= \frac{2y^2 - 2xy}{(2y - x)^3}$$