Math 141 Homework #8
Due Tuesday, 10/2/07
Extra Problems

These problems are taken from the Math 121 sample midterm exam from Fall 2005.

**Problem #1**  What value of \(x\) is \(f(x) = x^3 + \frac{1}{2}x^2 - 2x - 3\) decreasing most rapidly?

**Problem #2**  If \(c\) is a constant, then \(\lim_{h \to 0} \frac{e^{ch} - 1}{h}\) equals
(a) \(\ln c\)  (b) \(c\)  (c) \(e^c\)  (d) 0  (e) none of the above

**Problem #3**  Evaluate \(\lim_{h \to 4} \frac{h + 4}{\sqrt{h + 6} - \sqrt{2}}\) or explain why it does not exist.

**Problem #4**  Let \(f(x) = \frac{x - \sqrt{3}}{x^2 - 3}\). Evaluate the following:

(#4a) \(\lim_{x \to 1} f(x)\)
(#4b) \(\lim_{x \to 3} f(x)\)
(#4c) \(\lim_{x \to -\sqrt{3}} f(x)\)
(#4d) \(\lim_{x \to -2} f(x)\)

**Problem #5**  Suppose that \(f\) and \(g\) are functions such that \(f\) is continuous, \(f(-1) = 3\), and \(\lim_{x \to -1} \frac{g(x)}{f(x)^2 + 1} = 8\). Find \(\lim_{x \to -1} g(x)\).

**Problem #6**  Suppose that \(f(3) = 2, f'(3) = -1, g(3) = 3,\) and \(g'(3) = 5\). Find the following numbers:
(i) \((fg)'(3)\); (ii) \((g/f)'(3)\); (iii) the derivative of \(x^{-1}/f(x)\) at \(x = 3\).

**Problem #7**  Let \(f(x) = x^2 + 1\). Find every number \(a\) such that the line tangent to the graph of \(f(x)\) at the point \((a, f(a))\) passes through the point \((91, 0)\).
Problem #8  The position of a particle at time $t$ is given by $s(t) = t^3 - 4t^2 + 3t$ for $t \geq 0$.

(#8a) When is the velocity equal to 6?
(#8b) When is the acceleration equal to 0?
(#8c) When does the particle reverse its direction of motion?

Problem #9  Let $f$ be the function defined by $f(x) = 2x - 1$ for $x \geq 1$ and $f(x) = 3x - 2$ for $x < 1$. At $a = 1$, the function $f$ is

(a) continuous
(b) discontinuous because $\lim_{x \to 1^-} f(x)$ does not exist as a real number
(c) discontinuous because $\lim_{x \to 1} f(x) \neq f(1)$
(d) none of the above

Problem #10  Find an equation for the tangent line to the parametric curve $(x, y) = (3 \sin t, e^{2t})$ at the point $(0, 1)$.

Problem #11  Find an equation for the tangent line to the curve $3(x^2 + y^2)^2 = 14x^2 - y^2$ at the point $(\sqrt{2}, 1)$.

Problem #12  Calculate $\frac{d}{dx} [x^2 + 3]^{\sin x}$.

Problem #13  The derivative of $f(x) = \cos(x^2)$ at $x = 0$ is given by the expression

(a) $\lim_{h \to 0} \frac{\cos(h^2) - \cos h}{h}$
(b) $\lim_{h \to 0} \frac{\cos(h^2) - 1}{h^2}$
(c) $\lim_{h \to 0} \frac{\cos(h^2) - 1}{h}$
(d) $\lim_{h \to 0} \frac{\cos h - 1}{h}$
(e) none

Problem #14  For which value(s) of $c$ is the function $f(x)$ defined below continuous everywhere?

$$f(x) = \begin{cases} 
c^2x & \text{if } x \leq 1 \\
c + 6x & \text{if } x > 1
\end{cases}$$

Problem #15  Suppose that the tangent line to the graph of $f(x)$ at $(-1, 2)$ passes through the point $(1, 5)$. Find $f(-1)$ and $f'(-1)$.
**Problem #16** Suppose that \( f(3) = 2 \) and \( f'(3) = 5 \). Find the derivative of \((x^2 + 1)^{f(x)}\) at \( x = 3 \).

**Problem #17** Calculate the following limits, and for each one, draw a conclusion about an asymptote of some function.

\[
\text{(17a)} \quad \lim_{x \to -\infty} \frac{x^2 + \sqrt{3}x^3 + \sqrt{5}}{\sqrt{2} - 5x - \sqrt{2}x^3}
\]

\[
\text{(17b)} \quad \lim_{x \to \infty} \left(-2x + \sqrt{4x^2 - 3x + 1}\right)
\]

\[
\text{(17c)} \quad \lim_{x \to 3^-} \frac{x - 1}{(x - 3)(x - 4)}
\]

**Problem #18** Let \( f(x) = e^x \), \( g(x) = x - 3 \), and \( h(x) = 5x \). Find the functions \( f \circ g \), \( f \circ g \circ h \).

**Problem #19** Find a formula for the inverse of (i) the function \( g(t) = e^{1-t} + 3 \) with domain \((-\infty, \infty)\) and (ii) the function \( f(x) = \ln(x^2 + x - 1) \) with domain \((1, \infty)\).