Problem 1. Circle the correct answer.

(I) \( \ln(\ln(e^x)) \) is equal to
(A) \( \ln x \) (B) \( e^x \) (C) \( \ln e \) (D) \( e \) (E) None of the above.

(II) \( \sqrt{9e^{6x}} \) is equal to
(A) \( 3e^{\frac{3x}{2}} \) (B) \( 3 + e^{3x} \) (C) \( 3e^{3x} \) (D) \( (\sqrt{e})^{6x} \) (E) None of the above.

(III) \( \ln(x^4) + \ln 4 \) is equal to
(A) \( \ln(x^4 + 4) \) (B) \( 4 \ln x \) (C) \( 4 \ln(x + 4) \) (D) \( \ln(\frac{x}{4}) + \ln 4 \) (E) None of the above.

(IV) The inverse of the function \( f(x) = \ln(x) \) is
(A) \( g(x) = (\ln x)^{-1} \) (B) \( g(x) = e^x \) (C) \( g(x) = e^{-x} \) (D) \( g(x) = \ln(\frac{1}{x}) \) (E) None of the above.

Problem 2. Find the derivative of the following functions

(A) \( f(x) = \ln(x^3 + 1) \).

(B) \( f(x) = e^{x^2 - \sqrt{x}}. \)

(C) \( f(x) = e^x \ln(e^x + 1) \).

(D) \( f(x) = \cos(e^x) \).

(E) \( f(x) = \ln(\sin(x^2)) \).

(F) \( f(x) = \sin(\sin(\sqrt{x})) \).

Problem 3. Find the maximum and minimum values of the following function on the given intervals. Justify your work.

(A) \( f(x) = 2 + \sin(x), \quad x \in [0, 2\pi] \).

(B) \( f(x) = \sqrt{\sin x + 2}, \quad x \in [0, 2\pi] \).

(C) \( f(x) = x - \sin(x), \quad x \in [\pi/2, 3\pi/2] \).

(D) \( f(x) = 6(1 - \sin \frac{\pi x}{2}), \quad x \in [1/2, 5/4] \).

Problem 4. Find the coordinates of all relative extrema and inflection points of \( f(x) = \frac{x^2}{e^x} \).

Problem 5. Compute the following integrals. Check your answers.

(A) \( \int 2 \sin(2x) \, dx \).

(B) \( \int x(x^2 + 2x\pi) \, dx \).

(C) \( \int e^{4x} \, dx \).

(D) \( \int (3x^{3/2} + \frac{1}{x}) \, dx \).

(E) \( \int (4x - 12x^3)(2x^2 - 3x^4)^7 \, dx \).

Problem 6. An architect wishes to design a ramp inclined \( 30^\circ \) leading from the ground level to a second-story door in a parking garage. How far from the building must the ramp start if the garage is 12 feet above the ground level?

Problem 7. A patient’s temperature is 108 degrees and is changing at the rate of \( t^2 - 4t \) degrees per hour, where \( t \) is the number of hours since taking fever-reducing medication. Find the patient’s temperature after 2 hours.
Brief Solutions
(you may have to show more work in the exam)

Problem 1.
(I) \( \ln(\ln(e^x)) = \ln x \)  
(II) \( \sqrt{9e^{3x}} = 3e^{3x} \)  
(III) \( \ln\left(\frac{x}{4}\right) + \ln 4 = \ln x \)  
(IV) The inverse of the function \( f(x) = \ln(x) \) is \( g(x) = e^x \).

Problem 2.
(A) \( f'(x) = \frac{3x^2}{x^3+1} \).
(B) \( f'(x) = e^{e^{x}} - \sqrt{2}(2x - \frac{1}{2\sqrt{x}}) \).
(C) \( f'(x) = e^x \ln(e^x + 1) + \frac{e^{2x}}{e^x + 1} \).
(D) \( f'(x) = -\sin(e^x) e^x \).
(E) \( f'(x) = \frac{1}{\sin(\sqrt{x})} \cos(\sqrt{x}) 2x = 2x \cot(\sqrt{x}) \).
(F) \( f'(x) = \cos(\sin(\sqrt{x})) \cos(\sqrt{x}) \frac{1}{2\sqrt{x}} \).

Problem 3.
(A) \( f(3\pi/2) = 1 \) and \( f(\pi/2) = 3 \) (\( f \) is \( \sin x \) shifted two units up).
(B) \( f'(x) = \frac{\cos x}{2\sqrt{\sin x} + 2} \), \( f'(x) = 0 \) for \( x = \pi/2 \) or \( x = 3\pi/2 \). \( \text{Max=}f(\pi/2) = \sqrt{3} \), \( \text{Min=}f(3\pi/2)=1 \).
(C) \( f'(x) = 1 - \cos x \), \( f'(x) \) \( \neq 0 \) on \( [\pi/2, 3\pi/2] \). \( \text{Max=}f(3\pi/2) = 3\pi/2 + 1 \), \( \text{Min=}f(\pi/2) = \pi/2 - 1 \)
(D) \( \sin(x\pi/2) \) is positive on \([1/2, 5/4]\) and \( \sin(x\pi/2) = 1 \) there for \( x = 1 \). \( \text{Max=}f(1/2) = 6(1 - \sqrt{2}) \), \( \text{Min=}f(1) = 0 \).

Problem 4.
\( f'(x) = 2xe^{-x} - x^2e^{-x} = e^{-x}(2x - x^2) \)
\( f''(x) = -e^{-x}(2x - x^2) + e^{-x}(2 - 2x) = e^{-x}(x^2 - 4x + 2) \)
\( f'(x) = 0 \iff e^{-x}(2x - x^2) = 0 \iff (2x - x^2) = 0 \iff x = 0 \) or \( x = 2 \).
\( f''(0) > 0 \), then \( (0,0) \) is a local minimum.
\( f''(2) = -2e^{-2} < 0 \), then \( (2, 4e^{-2}) \) is a local maximum.
\( f''(x) = 0 \iff e^{-x}(x^2 - 4x + 2) = 0 \iff (x^2 - 4x + 2) = 0 \iff x = 2 \pm \sqrt{2} \)
\( f''(0) > 0 \) and \( f''(2) < 0 \), then \( (2 - \sqrt{2}, f(2 - \sqrt{2})) \) is an inflection point.
\( f''(2) < 0 \) and \( f''(4) > 0 \), then \( (2 + \sqrt{2}, f(2 + \sqrt{2})) \) is an inflection point.

Problem 5.
(A) \( -\cos 2x + C \).
(B) \( 1/4x^4 + 2/3x^3 + C \).
(C) \( e^{4x} + C \).
(D) \( (6/5)x^{5/2} + \ln|x| + C \).
(E) \( (1/18)(2x^2 - 3x^4)^{18} + C \)

Problem 6.
\( 30^\circ = \pi/6, 1/\sqrt{3} = \tan(\pi/6) = 12/x \) and hence \( x = 12\sqrt{3} \) feet.

Problem 7.
\( T(t) = 1/3t^3 - 2t^2 + C \).
\( T(0) = 108 \) so \( C = 108, T(2) = 1/32^3 - 2 \cdot 2^2 + 108 = 102.67 \) degrees.