The midterm exam is on Tuesday, March 16, 5:45p.m.–7:45p.m. – 1005 Haworth.

The exam will cover the following sections from the textbook:

- Sections S1.3, S1.4.

These problems are intended to be used as part of a review for the midterm exam. Do not confuse this, however, with a sample midterm. The problems below are not a substitute for studying all the material covered in class and the homework assignments. Review also the material for the first test.

**Problems from the textbook:**

- Sec. B2.3: 2, 14c), 17a).
- Sec. B2.4: 1, 8.
- Sec. B3.1: 3, 12.
- Sec. B3.2: 13, 25.
- Sec. B3.3: 2, 10.
- Sec. B3.4: 8, 10, 13, 17, 20, 27.
- Sec. B3.5: 4, 8, 17, 19.
- Sec. B3.6: 1, 7, 13, 15, 19, 23, 24, 27.

**Additional problems:**

1. Answer the following true or false.
   a) T F Let A, B be $n \times n$ invertible matrices. Then $(AB)^{-1} = A^{-1}B^{-1}$.
   b) T F A homogeneous system of three equations in four unknowns has no solution.
   c) T F Let $Ax = b$ be an $m \times n$ linear system. Then $b \in \mathbb{R}^n$.

2. Given $g(x, y) = \sqrt{x^2 + y^2}$, (a) Sketch the level curves of this function in the $xy$-plane, (b) sketch the graph of the function.

3. If $g, h : \mathbb{R}^2 \to \mathbb{R}$ and $f : \mathbb{R}^3 \to \mathbb{R}$ are functions with continuous partial derivatives and $F : \mathbb{R}^3 \to \mathbb{R}$ is defined by $F(s, t, u) = f(g(s, t), tu^2, h(s, u))$. Express $\partial F/\partial u$ in terms of the partial derivatives of the functions $f(x, y, z), g(s, t)$ and $h(s, u)$. 


4. Find (a) the Jacobian matrix of \( f(x, y) = (x^2y - e^y, x e^{xy}, e^{x^3}y^3) \) at \((1, -1)\) and (b) the Jacobian matrix of \( g(s, t, u) = (s^3tu^2 - su, st, t^u) \) at \((-1, 0, -1)\).

5. Given \( f : \mathbb{R}^2 \to \mathbb{R}^2 \) and \( g : \mathbb{R}^2 \to \mathbb{R}^2 \) such that \( f(1, 1) = (1, 0) \), \( J(g \circ f)(1, 1) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \) and \( Jg(1, 0) = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \), find \( Jf(1, 1) \).

6. The augmented matrix of a system of equations and its reduced row echelon form is given below.

\[
\begin{bmatrix}
1 & 2 & 0 & 4 & -5 \\
-3 & -6 & 1 & -9 & 22 \\
4 & 8 & 0 & 16 & -19
\end{bmatrix} \quad \rightarrow \quad \begin{bmatrix}
1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 3 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

a) Does the system have solutions?

b) Find all the solutions of the linear system

\[
\begin{bmatrix}
1 & 2 & 0 \\
-3 & -6 & 1 \\
4 & 8 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} =
\begin{bmatrix}
-4 \\
-9 \\
16
\end{bmatrix}.
\]

7. For what values of \( a \) is \( A = \begin{bmatrix} a - 2 & 2 \\ a - 2 & a + 2 \end{bmatrix} \) singular?

(A) 0, 2 \quad (B) -2, 4 \quad (C) 2, 4 \quad (D) 2 \quad (E) 0, -2

8. The matrix expression \((A + B)(A - B)\) can be rewritten as

(a) \(A^2 - B^2\) \quad (b) \(A^2 - AB + BA - B^2\) \quad (c) \(A^2 - BA + AB - B^2\) \quad (d) all of the above \quad (e) none of the above

9. Which of the following is a reduced row echelon form?

(A) \[
\begin{bmatrix}
1 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

(B) \[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0
\end{bmatrix}
\]

(C) \[
\begin{bmatrix}
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

(D) \[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

(E) none of the above

10. The figure below represents level sets of a function \( f : \mathbb{R}^2 \to \mathbb{R} \).
(a) At the point $P$, is $\frac{\partial f}{\partial y}$ positive or negative?

(b) Estimate $\frac{\partial f}{\partial x}$ at the point $Q = (2, 0)$. 