1. A point has position at time $t$ given by $\mathbf{x}(t) = (t, t^2, 1 + t^2)$, for $0 \leq t \leq 1$. At time $t = 1$ the point leaves this curve and flies off along the tangent line while maintaining the constant velocity attained at $t = 1$. Where is the point at $t = 2$?

2. Consider the surface given by the graph of the function $f(x, y) = 2y^2 - x^2y$.
   a) Find an equation of the tangent plane to the surface at the point $P(1, 1, 1)$.
   b) Find the equation of the normal line to the surface at the same point.
   c) A curve given by the vector valued function $\mathbf{x}(t)$ satisfies that $\mathbf{x}(1) = (1, 1, 1)$ and $\mathbf{x}'(1) = (4, 0, 1)$. Can a small arc of the curve be contained in the given surface for $t$ close to 1? Justify your answer.

3. A cyclist goes on a mountain road and, at time $t$, her position path is given by $\mathbf{x}(t) = (t, \frac{2}{3}t^3/2, 2 - t^2)$. On which of the following two mountains does her path lie?
   Mountain 1 has height $z = H_1(x, y) = 2 + \frac{9}{4}y^2 - x^3 - x^2$.
   Mountain 2 has height $z = H_2(x, y) = 2 + \frac{3}{2}y - x^2 - x^3$.

4. A particle is traveling in space with its position given by $\mathbf{x}(t) = t^2 \mathbf{i} + t \mathbf{j} + 2t^2 \mathbf{k}$.
   a) Find the velocity vector of the particle at time $t = 4$.
   b) Find the acceleration vector for the particle as a function of $t$.

5. A caterpillar is standing on a hill whose height is given by $H(x, y) = x^3 - xy$
   Take “North” to be the direction of the positive y-axis and “East” to be the positive x-axis and use this information to answer the following questions:
   a) If the caterpillar is standing at the point $(2, 1)$ and begins walking South, is it going uphill, downhill, or neither?
   b) If the caterpillar is standing at the point $(1, 1)$ and walks 1 units South, then 2 units West, what is the total change in elevation?
   c) If the caterpillar is standing at the point $(-1, 2)$, describe the direction (by a unit vector) that the caterpillar should move to go uphill fastest.

6. Circle only one of the given answers for each question.
   a) Let $f(x, y, z) = e^{x+y} \cos z$. Then $\nabla f(0, 0, \pi)$ is
      (A) $-\mathbf{i} - \mathbf{j}$   (B) $\mathbf{i} + \mathbf{j}$   (C) $\mathbf{i} - \mathbf{j} + \mathbf{k}$   (D) $\mathbf{i} + \mathbf{j} - \mathbf{k}$

   b) The tangent plane to the surface $x^4 - xy + z^2 = 1$ at $(0, 1, 1)$ is
      (A) $-x + 2z = 2$   (B) $y + z = 2$   (C) $-x + y + 2z = 3$   (D) $2z = 2$
c) The level surfaces of the function \( f(x, y, z) = x + 2y + z - 5 \) are
(A) planes perpendicular to \((1, 2, 1)\)  (B) planes perpendicular to \((1, 2, -5)\)
(C) concentric spheres  (D) none of the previous.

7. Suppose that the gradient \( \nabla f(2, 4) \) of a function \( f(x, y) \) has length 5. Is there a unit vector \( \mathbf{u} \) for which the directional derivative \( D_{\mathbf{u}} f \) at the point \((2, 4)\) is 7? Justify your answer.

8. Calculate the area of the triangle with vertices \((1, 2, 3), (4, -2, 1)\) and \((-3, 1, 0)\).

9. Give a set of parametric equations for the plane determined by \(2x + 3y - 5z = 30\).

10. Determine all second order partial derivatives of the function \( f(x, y) = \sin \sqrt{x^2 + y^2} \).

11. Compute the matrix of partial derivatives \( Df(a) \) where \( f(s, t) = (s^2, st, t^2) \) and \( a = (1, -1) \).

12. Compute the matrix of partial derivatives \( D(f \circ g) \), where \( f(x, y, z) = (x + y + z, xyz, e^z) \) and \( g(s, t) = (st, s + t, t^3) \).

13. Let \( f : \mathbb{R}^2 \to \mathbb{R}^2 \) and \( g : \mathbb{R}^2 \to \mathbb{R}^2 \) be two functions such that \( f(1, 1) = (1, 0) \), \( Df(1, 1) = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \), \( Df(1, 0) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \), \( Dg(1, 1) = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \), and \( Dg(1, 0) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \). Then,

\[
D(g \circ f)(1, 1) =
\]

(A) \( \begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix} \)  (B) \( \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \)  (C) \( \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \)  (D) \( \begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix} \)

(E) None of the above.

14. Mark all the correct answers. Let \( f(x, y) \) be a function of two variables. Then, \( \frac{\partial f}{\partial x}(x_0, y_0) \) is equal to
(A) \( \frac{\partial f}{\partial y}(x_0, y_0) \).
(B) the rate of change of \( f \) with respect to \( x \) at the point \((x_0, y_0)\) when \( y = y_0 \) is fixed.
(C) the slope of the tangent line to the curve of intersection of the plane \( y = y_0 \) with the graph of \( z = f(x, y) \) at the point \((x_0, y_0)\).
(D) all of the above.

15. Write an equation for the tangent plane to the surface given by the level set corresponding to the value 3 of the function \( F : \mathbb{R}^3 \to \mathbb{R}, \) \( F(x, y, z) = y^4x + z^2y + zx^3 \), at the point \((1, 1, 1)\).