

**MATH 540: PRACTICE PROBLEMS FOR EXAM 2**

1. Solve the following system of congruences:  $x \equiv 15 \pmod{6}$ ,  $x \equiv 2 \pmod{7}$ ,  $x \equiv 9 \pmod{11}$ .
2. For  $\phi(n)$ , the Euler  $\phi$  function:
  - (a) Calculate  $\phi(1512)$ .
  - (b) Verify Euler's product formula for  $n = 1512$ .
  - (c) Verify that  $\phi(81 \cdot 77) = \phi(81) \cdot \phi(77)$ .
3. Calculate  $(3451)^{8,888,213} \pmod{13}$ .
4. Use Euler's theorem to find the quadratic residues modulo 19.
5. Calculate the following values of the Legendre symbols:  $\left(\frac{6}{241}\right)$ ,  $\left(\frac{1,000,001}{17}\right)$ ,  $\left(\frac{-48}{101}\right)$ .
6. Calculate  $\left(\frac{5}{31}\right)$ , using Gauss's Lemma.
7. For the polynomial  $f(x) = 20x^2 - 7x + 77$ :
  - (a) Use the law of quadratic reciprocity to determine if  $f(x)$  has a root modulo  $p = 7$ .
  - (b) Find the all integers  $u$  such that  $u$  is a root of  $f(x)$  modulo 19.