1. Prove the following two statements by induction:
   (a) \( \sum_{k=1}^{n} (-1)^k k^2 = \frac{(-1)^n n(n+1)}{2} \).
   (b) \( \sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1} \).

2. Let \( d_1 = \frac{2}{3} \) and \( d_2 = \frac{3}{5} \). For \( n \geq 3 \), set \( d_n := d_{n-1} \cdot d_{n-2} \). Use the Well Ordering Principle to show that \( d_n < 1 \), for all \( n \geq 1 \).

3. Use Let \( a = \sigma(24) \) and \( b = \tau(24) \). Here \( \sigma(24) \) means the sum of the divisors of 24 and \( \tau(24) \) means the number of divisors of 24. Use the division algorithm to find \( \text{GCD}(a, b) \). Then use Bezout’s Principle to write \( \text{GCD}(a, b) \) as a combination of \( a \) and \( b \). Finally, find \( \text{LCM}(a, b) \).

4. Use the Fundamental Theorem of arithmetic to find the \( \text{GCD} \) and \( \text{LCM} \) of 63,000 and 36,690.

5. Consider the relation on \( \mathbb{Z}^+ \times \mathbb{Z}^+ \) given by \( (a, b) \sim (c, d) \) if and only if \( a + d = b + c \).
   (a) Prove that \( \sim \) is an equivalence relation.
   (b) Describe the equivalence class of \((3, 5)\).
   (c) Let \( X \) be the set of equivalence classes of \( \sim \). Define \( f : X \to \mathbb{Z} \), by \( f([a, b]) = a - b \). Prove that \( f \) is well defined, in other words, the value of \( f \) does not change if we use a different representative for the class \([a, b]\).

6. For \( n \geq 2 \), let \( \mathbb{Z}_n \) denote the integers modulo \( n \).
   (a) Write out addition and multiplication tables for \( \mathbb{Z}_7 \).
   (b) Can you explain why every non-zero element of \( \mathbb{Z}_7 \) has a multiplicative inverse?
   (c) Find a solution to the congruence \( 7x \equiv 5 \pmod{12} \).
   (d) Find an integer \( n \) so that the congruence \( 7x \equiv 5 \pmod{n} \) does not have a solution.