

MATH 540: STUDY GUIDE FOR EXAM I

The following is a list of topics and types of problems you should know for Exam I.

1. Mathematical Induction. Be able to apply mathematical induction to prove an elementary statement.
2. The Well Ordering Principle. Know how to state it and how to use it.
3. The Division Algorithm and its consequences. Be able to apply the division algorithm to find the GCD of two positive integers and use it to find the coefficients required in Bezout's principle.
4. Know the relationship between the LCM and GCD of two positive integers.
5. Know the Fundamental Theorem of Arithmetic and its consequences, especially in regards to finding LCMs and GCDs.
6. Be able to verify that a given relation is an equivalence relation.
7. Know the formulas for $\tau(n)$, the number of divisors of n , and $\sigma(n)$, the sum of the divisors of n , and how to use them.
8. Know basic properties of and how to compute with integers modulo n , including solving simple linear equations.
9. Be able to reproduce the proof of any one of the following three theorems:
 - (i) Every positive integer can be written as a product of prime numbers, i.e., the existence part of the Fundamental Theorem of Arithmetic.
 - (ii) If p is a prime number and $p|ab$, then $p|a$ or $p|b$.
 - (iii) There are infinitely many prime numbers of the form $4n - 1$.