The **Method of Markers** is a fair-division method for a multiplayer game with discrete goods (e.g., Halloween candy).

In comparison to the Method of Sealed Bids:

- Everyone gets at least (roughly) a fair share, provided they “bid” honestly.

- **Disadvantage**: Not suitable if the goods have widely varying values (e.g., an estate)

- **Advantage**: Doesn’t require players to put in cash
In Example 3.11 on pp. 98–100 of Tannenbaum, four kids (Alice, Bianca, Carla, Dana) have a pile of Halloween candy that they need to share fairly.
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**Step 0: Arranging.** Line up the booty to be divided in a single row.
In Example 3.11 on pp. 98–100 of Tannenbaum, four kids (Alice, Bianca, Carla, Dana) have a pile of Halloween candy that they need to share fairly.

**Step 0: Arranging.** Line up the booty to be divided in a single row.
Step 1: Bidding.
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Alice places three markers $A_1$, $A_2$, $A_3$, dividing the row into four segments, each of which she thinks is a fair share.
The Method of Markers: Step 1

That is, Alice would consider any one of these shares to be fair:

1, 2, 3, 4
(Alice's 1st segment)

6, 7, 8, 9, 10, 11
(2nd segment)

12, 13, 14, 15, 16
(3rd segment)

17, 18, 19, 20
(4th segment)
The Method of Markers: Step 1

That is, Alice would consider any one of these shares to be fair:

- \{1, 2, 3, 4, 5\} (Alice’s 1st segment)
- \{6, 7, 8, 9, 10, 11\} (2nd segment)
- \{12, 13, 14, 15, 16\} (3rd segment)
- \{17, 18, 19, 20\} (4th segment)
The Method of Markers: Step 1

That is, Alice would consider any one of these shares to be fair:

\[
\begin{align*}
\{1, 2, 3, 4, 5\} & \quad \text{(Alice’s 1st segment)} \\
\{6, 7, 8, 9, 10, 11\} & \quad \text{(2nd segment)} \\
\{12, 13, 14, 15, 16\} & \quad \text{(3rd segment)} \\
\{17, 18, 19, 20\} & \quad \text{(4th segment)}
\end{align*}
\]
Similarly, Bianca places markers $B_1$, $B_2$, $B_3$; Carla places markers $C_1$, $C_2$, $C_3$; Dana places markers $D_1$, $D_2$, $D_3$. 
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In order to keep the method fair, the players must all place their markers at the same time. (For example, they can submit sealed envelopes with the positions of their markers.)
The Method of Markers: Step 1

There are a total of 12 markers (3 for each of 4 players).

Now what?
Step 2: Allocations.
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Step 2.1: Locate the leftmost 1st marker ($A_1$, $B_1$, $C_1$, or $D_1$). (If there is a tie, choose one randomly.)
Step 2: Allocations.

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The corresponding player (here, Bianca) gets her 1st segment.

— In this case, Bianca gets \([1 \ 2 \ 3 \ 4]\)
Step 2: Allocations.

**Step 2.1:** Locate the leftmost 1st marker ($A_1$, $B_1$, $C_1$, or $D_1$). (If there is a tie, choose one randomly.)

The corresponding player (here, Bianca) gets her 1st segment.

In this case, Bianca gets 1 2 3 4

Then, remove all Bianca’s markers.
The Method of Markers: Step 2

Bianca’s share
The Method of Markers: Step 2

Removing Bianca’s markers
Step 2: Allocations.

Step 2.2: Locate the leftmost second marker ($A_2$, $C_2$, or $D_2$).
Step 2: Allocations.

Step 2.2: Locate the leftmost second marker ($A_2$, $C_2$, or $D_2$).

The corresponding player (here, Carla) gets her 2nd segment.

— Carla’s share: 7 8 9
Step 2: Allocations.

**Step 2.2:** Locate the leftmost second marker ($A_2$, $C_2$, or $D_2$).

The corresponding player (here, Carla) gets her 2nd segment.

— Carla’s share: 7 8 9

Then, remove all Carla’s markers.
The Method of Markers: Step 2

Carla’s share (focusing on 1st and 2nd markers)
The Method of Markers: Step 2

Removing Carla’s markers
The Method of Markers: Step 2

Note that 5 and 6 have not been allocated to anyone yet.
Step 2: Allocations.

Step 2.3: Locate the leftmost third marker ($A_3$ or $D_3$).
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This time, there is a tie; let’s say a coin toss chooses Alice rather than Dana.
Step 2: Allocations.

Step 2.3: Locate the leftmost third marker ($A_3$ or $D_3$).

This time, there is a tie; let’s say a coin toss chooses Alice rather than Dana.

Alice gets her 3rd segment.

Alice’s share: 12 13 14 15 16
Step 2: Allocations.

Step 2.3: Locate the leftmost third marker ($A_3$ or $D_3$).

This time, there is a tie; let’s say a coin toss chooses Alice rather than Dana.

Alice gets her 3rd segment.

— Alice’s share: 12 13 14 15 16

Then, remove all Alice’s markers.
The Method of Markers: Step 2
Step 2: Allocations.

Step 2.4: The last player left gets her last segment.

— Dana’s share: 17 18 19 20
The Method of Markers: Step 2

5  6
m&m's  NutRageous

10  11
NutRageous  Baby Ruth

17  18  19  20
NutRageous  Snickers  Mr. Goodbar  Baby Ruth

$D_3$ to Dana
At this point, everyone has been given a fair share, and usually, there is candy left over!

Step 3: Divide the surplus. (It doesn’t matter how — for instance, players get to pick one item at a time in a random order.)
The Method of Markers: Review

**Step 0:** Arrange the booty to be divided in a row.

**Step 1:** Bidding. Each of the $N$ players bids by placing $N - 1$ markers to separate the booty into $N$ fair shares (in that players’s opinion).

(Preserve the Privacy Assumption by having players reveal their bids simultaneously.)
Step 2: Allocation.

- Locate the leftmost 1st marker.
  Give that player his 1st segment (i.e., from the left end to his 1st marker). Then remove all his markers.
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- Locate the leftmost 1st marker.
  Give that player his 1st segment (i.e., from the left end to his 1st marker). Then remove all his markers.

- Locate the leftmost 2nd marker.
  Give that player her 2nd segment (i.e., between her 1st and 2nd markers). Then remove all her markers.
The Method of Markers: Review

**Step 2: Allocation.**

- Locate the leftmost 1st marker. Give that player his 1st segment (i.e., from the left end to his 1st marker). Then remove all his markers.

- Locate the leftmost 2nd marker. Give that player her 2nd segment (i.e., between her 1st and 2nd markers). Then remove all her markers.

- Locate the leftmost 3rd marker. Give that player her 3rd segment (i.e., between his 2nd and 3rd markers). Then remove all his markers.

...
The Method of Markers: Review

**Step 2: Allocation.**

- Eventually, locate the leftmost \((N - 1)^{st}\) marker. Give that player her \((N - 1)^{st}\) segment (i.e., between her \((N - 2)^{nd}\) and \((N - 1)^{st}\) markers). Then remove all her markers.

- The last player gets his last segment (i.e., from his \((N - 1)st\) marker to the end of the row).
Step 3: Division of Surplus.

If there are items left over, divide them by, e.g., taking turns choosing one
(or, if there are a lot of items, use the Method of Markers all over again!)
Everyone gets at least a fair share: the method is set up so that the segments allocated never overlap.
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The players have to be able to divide the booty into roughly equal shares.

The method works best if the goods are roughly equivalent in value to each other, and if the players’ preferences are fairly close.

(In the example above, what if one of the players is allergic to peanuts?)
And now for something completely different.
In 1735, the city of Königsberg (present-day Kaliningrad) was divided into four districts by the Pregel River.

The four districts were connected by seven bridges.
The Seven Bridges of Königsberg

Is it possible to take a walking tour of Königsberg in which you cross each of the seven bridges exactly once?
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